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Research Article

Some Results on Wijsman Ideal Convergence in Intuitionistic Fuzzy Metric Spaces

Ayhan Esi,[1] **Vakeel A. Khan**,[2] **Mobeen Ahmad**,[2] **and Masood Alam**[3]

[1]*Malatya Turgut Ozal University Engineering Faculty Dept. of Basic Engineering Sciences Malatya, Malatya 44040, Turkey*
[2]*Department of Mathematics, Aligarh Muslim University, Aligarh 202002, India*
[3]*Department of Mathematics and IT Center for Preparatory Studies, Sultan Qaboos University, P. O. Box 162 PC 123 Muscat, Oman*

Correspondence should be addressed to Ayhan Esi; aesi23@hotmail.com

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In the present work, we study and extend the notion of Wijsman $\mathcal{J}$–convergence and Wijsman $\mathcal{J}^*$–convergence for the sequence of closed sets in a more general setting, i.e., in the intuitionistic fuzzy metric spaces (briefly, IFMS). Furthermore, we also examine the concept of Wijsman $\mathcal{J}^*$–Cauchy and $\mathcal{J}$–Cauchy sequence in the intuitionistic fuzzy metric space and observe some properties.

1. Introduction

In 1951, Fast [1] initiated the theory of statistical convergence. It is an extremely effective tool to study the convergence of numerical problems in sequence spaces by the idea of density. Statistical convergence of the sequence of sets was examined by Nuray and Rhoades [2]. Ulusu and Nuray [3] studied the Wijsman lacunary statistical convergence sequence of sets and connected with the Wijsman statistical convergence. Esi et al. [4] introduced the Wijsman λ-statistical convergence of interval numbers. Kostyrko et al. [5] generalized the statistical convergence and introduced the notion of ideal $\mathcal{J}$–convergence. Salát et al. [6, 7] investigated it from the sequence space viewpoint and associated with the summability theory. Further, it was analyzed by Khan et al. [8] with the help of a bounded operator. In 2008, Das et al. [9] analyzed $\mathcal{J}$ and $\mathcal{J}^*$–convergence for double sequences. Kisi and Nuray [10] initiated new convergence definitions for the sequence of sets. Furthermore, Gümüş [11] studied the Wijsman ideal convergent sequence of sets using the Orlicz function.

In 1965, Zadeh [12] started the fuzzy sets theory. This theory has proved its usefulness and ability to solve many problems that classical logic was unable to handle. Karmosil et al. [13] introduced the fuzzy metric space, which has the most significant applications in quantum particle physics. Afterward, numerous researchers have studied the concept of fuzzy metric spaces in different ways. George et al. [14, 15] modified the notion of fuzzy metric space and determined a Hausdorff topology for fuzzy metric spaces. Atanassov [16] generalized the fuzzy sets and introduced the notion of intuitionistic fuzzy sets in 1986. Park [17] examined the notion of IFMS, and Saadati and Park [18] further analyzed the intuitionistic fuzzy topological spaces. Moreover, statistical convergence, ideal convergence, and different properties of sequences in intuitionistic fuzzy normed spaces were examined by Mursaleen et al. [19–21]. Also one can refer to Sengül and Et [22], Sengül et al. [23], Et and Yilmazer [24], Mohiuddine and Alamri [25], and Mohiuddine et al. [26, 27].

2. Preliminaries

We recall some concepts and results which are needed in sequel.

Definition 1 [5]. A family of subsets $\mathcal{J} \subseteq 2^{\mathbb{N}}$ is known as an ideal in a non-empty set $\mathbb{N}$, *if*

(1) $\varnothing \in \mathcal{J}$,

(2) for any $\mathcal{C}, \mathcal{D} \in \mathcal{J} \Rightarrow \mathcal{C} \cup \mathcal{D} \in \mathcal{J}$,

(3) for any $\mathcal{C} \in \mathcal{J}$ and $\mathcal{D} \subseteq \mathcal{C}, \Rightarrow \mathcal{D} \in \mathcal{J}$.

Remark 2 [5]. An ideal $\mathcal{J}$ is known as non-trivial if $\mathbb{N} \notin \mathcal{J}$. A nontrivial ideal $\mathcal{J}$ is known as admissible if $\{\{n\}: n \in \mathbb{N}\} \in \mathcal{J}$.

Definition 3 [5]. A nonempty subset $\mathcal{F} \subseteq 2^{\mathbb{N}}$ is known as filter in $\mathbb{N}$ if

(1) for every $\emptyset \notin \mathcal{F}$,

(2) for every $\mathcal{C}, \mathcal{D} \in \mathcal{F} \Rightarrow \mathcal{C} \cap \mathcal{D} \in \mathcal{F}$,

(3) for every $\mathcal{C} \in \mathcal{F}$ with $\mathcal{C} \subseteq \mathcal{D}$, one obtain $\mathcal{D} \in \mathcal{F}$.

Proposition 4 [5]. *For every ideal $\mathcal{J}$, there is a filter $\mathcal{F}(\mathcal{J})$ associated with $\mathcal{J}$ defined as follows:*

$$\mathcal{F}(\mathcal{J}) = \{K \subseteq \mathbb{N} : K = \mathbb{N} \setminus A, \textit{for some } A \in \mathcal{J}\}. \tag{1}$$

Definition 5 [5]. Let $\{\mathcal{C}_1, \mathcal{C}_1, \cdots\}$ be a mutually disjoint sequence of sets of $\mathcal{J}$. Then, there is sequence of sets $\{\mathcal{D}_1, \mathcal{D}_1, \cdots\}$ so that $\cup_{j=1}^{\infty} \mathcal{D}_j \in \mathcal{J}$ and each symmetric difference $\mathcal{C}_j \triangle \mathcal{D}_j (j = 1, 2, \cdots)$ is finite. In this case, admissible ideal $\mathcal{J}$ is known as property (AP).

Lemma 6 [28]. *Suppose $\mathcal{J}$ be an admissible ideal alongside property (AP). Let a countable collection of subsets $\{\mathcal{C}_k\}_{k=1}^{\infty}$ of positive integer $\mathbb{N}$ in such a way that $\mathcal{C}_k \in \mathcal{F}(\mathcal{J})$. Then, there exists a set $\mathcal{C} \subset \mathbb{N}$ such that $\mathcal{C} \setminus \mathcal{C}_k$ is finite for all $k \in \mathbb{N}$ and $\mathcal{C} \in \mathcal{F}(\mathcal{J})$.*

Definition 7 [29]. Let $(\mathcal{M}, d)$ be a metric space and $\{\mathcal{C}_k\}$ be a sequence of nonempty closed subsets of $\mathcal{M}$ which is said to be Wijsman convergent to the closed $\mathcal{C}$ *of* $\mathcal{M}$, if

$$\lim_{k\to\infty} d(x, \mathcal{C}_k) = d(x, \mathcal{C}) \text{for every} \in \mathcal{M}. \tag{2}$$

In other words, $W - \lim_{k\to\infty} \mathcal{C}_k = \mathcal{C}$.

In 2012, Nuray and Rhoades [2] initiated the theory of Wijsman statistical convergence for a sequence of sets. Furthermore, Kisi and Nuray [10] extended it into $\mathcal{J}$ -convergence.

Definition 8 [10]. Suppose $(\mathcal{M}, d)$ is a metric space. A nonempty closed subset $\{\mathcal{C}_k\}$ of $\mathcal{M}$ is known as Wijsman $\mathcal{J}$ –convergent to a closed set $\mathcal{C}$, if for every $x \in \mathcal{M}$, one has

$$\{k \in \mathbb{N} : |d(x, \mathcal{C}_k) - d(x, \mathcal{C})| \geq \epsilon\} \in \mathcal{J}. \tag{3}$$

Hence, one writes $\mathcal{J}_W - \lim_{k\to\infty} \mathcal{C}_k = \mathcal{C}$.

Definition 9 [10]. Suppose $(\mathcal{M}, d)$ *is a metric space. A nonempty closed subset $\{\mathcal{C}_k\}$ of $\mathcal{M}$ is known as Wijsman $\mathcal{J}$ –Cauchy if for each $x \in \mathcal{M}$, there exists a positive integer $m = m(\epsilon)$ so that the set*

$$\left\{k \in \mathbb{N} : \left|d(x, \mathcal{C}_k) - d\left(x, \mathcal{C}_p\right)\right| \geq \epsilon\right\} \in \mathcal{J}, \text{for all } p \geq m. \tag{4}$$

Definition 10 [10]. Suppose $(\mathcal{M}, d)$ is a separable metric space and $\{\mathcal{C}_k\}, \mathcal{C}$ is nonempty closed subsets of $\mathcal{M}$. A sequence $\{\mathcal{C}_k\}$ is known as Wijsman $\mathcal{J}^*$ –convergent to $\mathcal{C}$ if and only if $\exists P \in \mathcal{F}(\mathcal{J})$ and $P = \{p = (p_j < p_{j+1}, j \in \mathbb{N})\} \subset \mathbb{N}$ in such a manner that

$$\lim_{k\to\infty} d\left(x, \mathcal{C}_{m_k}\right) = d(x, \mathcal{C}), \text{for every } x \in \mathcal{M}. \tag{5}$$

One writes $\mathcal{J}_W^* - \lim_{k\to\infty} \mathcal{C}_k = \mathcal{C}$.

Definition 11 [10]. Suppose $(\mathcal{M}, d)$ is a separable metric space and $\mathcal{J}$ is an admissible ideal. A sequence $\{\mathcal{C}_k\}$ of nonempty closed subsets of $\mathcal{M}$ is known as the Wijsman $\mathcal{J}^*$ –Cauchy sequence if there exists $P \in \mathcal{F}(\mathcal{J})$, where $P = \{p = (p_j < p_{j+1}, i \in \mathbb{N})\}$ in such a way that subsequence $\mathcal{C}_P = \{\mathcal{C}_{p_k}\}$ is Wijsman Cauchy in $\mathcal{M}$, i.e.,

$$\lim_{k,l\to\infty} |d\left(x, \mathcal{C}_{m_k}\right) - d\left(x, \mathcal{C}_{p_l}\right)| = 0. \tag{6}$$

Remark 12 [10]. In general, the Wijsman topology is not first-countable, if sequence of nonempty sets $\{\mathcal{C}_k\}$ is Wijsman convergent to set $\mathcal{C}$, then every subsequence of $\{\mathcal{C}_k\}$ may not be convergent to $\mathcal{C}$. Every subsequence of the convergent sequence $\{\mathcal{C}_k\}$ converges to the same limit provided that $\mathcal{M}$ is a separable metric space.

Definition 13 [17]. Let $\mathcal{M}$ be a nonempty set, η and φ be fuzzy sets on $\mathcal{M}^2 \times (0,\infty)$, $*$ be a continuous t-norm, and $\diamond$ be a continuous t -conorm. Then, the five-tuple $(\mathcal{M}, \eta, \varphi, *, \diamond)$ is known as an intuitionistic fuzzy metric space (for short, IFMS) if it fulfills the subsequent conditions for all $s, t > 0$ and for every $y, z, w \in \mathcal{M}$:

(a) $\eta(y, z, s) + \varphi(y, z, s) \leq 1$,

(b) $\eta(y, z, s) > 0$,

(c) $\eta(y, z, s) = 1$ if and only if $y = z$,

(d) $\eta(y, z, s) = \eta(z, y, s)$,

(e) $\eta(y, z, s) * \eta(z, w, t) \leq \eta(y, w, s + t)$,

(f) $\eta(y, z,.)$: $(0,\infty) \to 0, 1]$ is continuous,

(g) $\varphi(y, z, s) < 1$,

(h) $\varphi(y, z, s) = 0$ if and only if $y = z$,

(i) $\varphi(y, z, s) = \varphi(z, y, s)$,

(j) $\varphi(y, z, s) \diamond \varphi(z, w, t) \geq \varphi(y, w, s + t)$,

(k) $\varphi(y,.)$: $(0,\infty) \to 0, 1]$ is continuous.

In such situation, (η, φ) is called the intuitionistic fuzzy metric (briefly, IFM).

Example 14 [17]. Suppose $(\mathcal{M}, d)$ is a metric space. Define $c \diamond d = \min(c + d, 1)$ and $c * d = cd$ for all $c, d \in [0, 1]$, and suppose η and φ are fuzzy sets on $\mathcal{M}^2 \times (0,\infty)$ defined as

$$\eta(y,z,s)=\frac{s}{s+d(y,z)},\varphi(y,z,s)=\frac{d(y,z)}{s+d(y,z)}. \quad (7)$$

Then $(\mathscr{M},\eta,\varphi,*,\diamond)$ is an IFMS.

Definition 15 [18]. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be an IFMS and $\mathscr{C}$ be a nonempty subset of $\mathscr{M}$. For all $s>0$ and $x\in\mathscr{M}$, we define

$$\eta(x,\mathscr{C},s)=\sup\{\eta(x,y,s)\colon y\in\mathscr{C}\} \quad (8)$$

and

$$\varphi(x,\mathscr{C},s)=\inf\{\varphi(x,y,s)\colon y\in\mathscr{C}\}, \quad (9)$$

where $\eta(x,\mathscr{C},s)$ and $\varphi(x,\mathscr{C},s)$ are the degree of nearness and nonnearness of x to $\mathscr{C}$ at s, respectively.

Saadati and Park [18] studied the notion of convergence sequence with respect to IFMS which are defined as follows:

Definition 16 [18]. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be an IFMS. A sequence $x=(x_k)$ is convergent to ξ if for any $0<\epsilon<0$ *and* $s>0$ there exists $k_0\in\mathbb{N}$ in such a way that

$$\eta(x_k,\xi,s)>1-\epsilon \text{ and } \varphi(x_k,\xi,s)<\epsilon \text{ for all } k\geq k_0. \quad (10)$$

Definition 17 [20]. An IFMS $(\mathscr{M},\eta,\varphi,*,\diamond)$ is known as separable if it contains a countable dense subset, i.e., there is a countable set $\{x_k\}$ along with subsequent property: for any $s>0$ and for all $\xi\in\mathscr{M}$, there is at least one x_n in order that

$$\eta(x_n,\xi,s)\geq 1-\epsilon \text{ and } \varphi(x_n,\xi,s)\leq\epsilon, \text{ for each } \epsilon\in(0,1). \quad (11)$$

3. Wijsman $\mathscr{J}$ and $\mathscr{J}^*$–convergence in IFMS

Throughout this section, we denote $\mathscr{J}$ to be the admissible ideal in $\mathbb{N}$. We begin with the following definitions as follows.

Definition 18. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be an IFMS. A sequence of sets $\{C_k\}$ is said be Wijsman convergent to $\mathscr{C}$ if for every $\epsilon>0$ and $s>0$ there exists $k_0\in\mathbb{N}$ such that

$$\lim_{k\to\infty}\eta(x,C_k,s)=\eta(x,\mathscr{C},s) \text{ and } \lim_{k\to\infty}\varphi(x,C_k,s) = \varphi(x,C,s) \text{ for all } k\geq k_0. \quad (12)$$

The set of all Wijsman limit point of the sequence $\{\mathscr{C}_k\}$ is denoted by $L_{\{\mathscr{C}_k\}}$.

Definition 19. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be an IFMS and $\mathscr{J}$ be a proper ideal in $\mathbb{N}$. A sequence $\{\mathscr{C}_k\}$ of nonempty closed subsets of $\mathscr{M}$ is known as Wijsman $\mathscr{J}$ –convergent to $\mathscr{C}$ with respect to IFM (η,φ), if for every $0<\epsilon<1$, for each $x\in\mathscr{M}$ and for all $s>0$ such that

$$\{k\in\mathbb{N}:|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C},s)| \leq 1-\epsilon \text{ or } |\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C},s)|\geq\epsilon\}\in\mathscr{J}. \quad (13)$$

We write $(\eta,\varphi)-\mathscr{J}_W-\lim_{k\to\infty}\mathscr{C}_k=\mathscr{C}$.

Example 20. Suppose $(\mathscr{M},\eta,\varphi,*,\diamond)$ is an IFMS and $\mathscr{C},\{\mathscr{C}_k\}$ is nonempty closed subsets of $\mathscr{M}$. Assume $\mathscr{M}=\mathbb{R}^2$ and $\{\mathscr{C}_k\}$ are sequence defined by

$$\mathscr{C}_k=\begin{cases}(x,y)\in\mathbb{R}^2: 0\leq x\leq k, 0\leq y\leq \frac{1}{k}.x, \text{ if } k\neq n^2\\ (x,y)\in\mathbb{R}^2: x\geq 0, y=1, \text{ if } k=n^2,\end{cases}$$
$$\mathscr{C}=\{(x,y)\in\mathbb{R}^2: x\geq 0, y=0\}. \quad (14)$$

Since

$$\lim_{k\to\infty}\frac{1}{k}|\{n\leq k:|\eta((x,y),\mathscr{C}_k,s)-\eta((x,y),\mathscr{C},s)| \leq 1-\epsilon \text{ or } |\varphi((x,y),\mathscr{C}_k,s)-\varphi((x,y),\mathscr{C},s)|\geq\epsilon\}|=0. \quad (15)$$

Therefore, the sequence of sets $\{\mathscr{C}_k\}$ is Wijsman statistical convergent to the set $\mathscr{C}$.

Now, define the set S as

$$S(\epsilon)=\{k\in\mathbb{N}:|\eta((x,y),\mathscr{C}_k,s)-\eta((x,y),\mathscr{C},s)| \leq 1-\epsilon \text{ or } |\varphi((x,y),\mathscr{C}_k,s)-\varphi((x,y),\mathscr{C},s)|\geq\epsilon\}. \quad (16)$$

If we assume $\mathscr{J}=\mathscr{J}_d$, then the Wijsman statistical convergence coincides with the Wijsman ideal convergence. Therefore,

$$\{k\in\mathbb{N}:|\eta((x,y),\mathscr{C}_k,s)-\eta((x,y),\mathscr{C},s)| \leq 1-\epsilon \text{ or } |\varphi((x,y),\mathscr{C}_k,s)-\varphi((x,y),\mathscr{C},s)|\geq\epsilon\} =\{k\in\mathbb{N}:k=n^2\}\subset\mathscr{J}_d. \quad (17)$$

Definition 21. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be a separable IFMS and $\mathscr{J}$ be an admissible ideal in $\mathbb{N}$. A sequence $\{\mathscr{C}_k\}$ of nonempty closed subsets of $\mathscr{M}$ is known as Wijsman $\mathscr{J}$–Cauchy with respect to IFM (η,φ), if for each $0<\epsilon<1$, for each $x\in\mathscr{M}$ and for all $s>0$, $\exists\ l=l(\epsilon)$ such that

$$\{k\in\mathbb{N}:|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_l,s)| \leq 1-\epsilon \text{ or } |\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_l,t)|\geq\epsilon\}\in\mathscr{J}. \quad (18)$$

Definition 22. Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be a separable IFMS and $\{\mathscr{C}_k\}$ be any nonempty closed subset of $\mathscr{M}$. The sequence $\{\mathscr{C}_k\}$ is known as Wijsman $\mathscr{J}^*$–Cauchy with respect to IFM (η,φ), if there exists $P=\{p=(p_j)\colon p_j<p_{j+1}, j\in\mathbb{N}\}\subset\mathbb{N}$ and $P\in\mathscr{F}(\mathscr{J})$ with the result that the subsequence $\mathscr{C}_P=\{\mathscr{C}_{p_k}\}$ is Wijsman Cauchy in $\mathscr{M}$, i.e.

$$\lim_{k,l\to\infty} |\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta\left(x, \mathscr{C}_{p_l}, s\right)| = 1 \tag{19}$$

and

$$\lim_{k,l\to\infty} |\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi\left(x, \mathscr{C}_{p_l}, s\right)| = 0. \tag{20}$$

Definition 23. Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS and $\mathscr{J}$ be an proper ideal in $\mathbb{N}$. Let $\{\mathscr{C}_k\}$ be nonempty closed subsets of $\mathscr{M}$. The sequence $\{\mathscr{C}_k\}$ is known as Wijsman $\mathscr{J}^*$–convergent to $\mathscr{C}$ with respect to (η, φ), if there exists $P \in \mathscr{F}(\mathscr{J})$, where $P = \{p = (p_j) \colon p_j < p_{j+1}, j \in \mathbb{N}\} \subset \mathbb{N}$ such that for each $s > 0$, we have

$$\lim_{k\to\infty} \eta\left(x, \mathscr{C}_{p_k}, s\right) = \eta(x, \mathscr{C}, s), \tag{21}$$

and

$$\lim_{k\to\infty} \varphi\left(x, \mathscr{C}_{p_k}, s\right) = \varphi(x, \mathscr{C}, s). \tag{22}$$

In such case, we write $(\eta, \varphi) - \mathscr{J}^*_W - \lim \mathscr{C}_k = \mathscr{C}$.

In the following theorem, we prove that every Wijsman $\mathscr{J}$–convergent implies the Wijsman $\mathscr{J}$–Cauchy condition in IFMS:

Theorem 24. *Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS and let $\mathscr{J}$ be an arbitrary admissible ideal. Then, every Wijsman $\mathscr{J}$–convergent sequence of closet sets $\{\mathscr{C}_k\}$ is Wijsman $\mathscr{J}$ –Cauchy with respect to IFM (η, φ).*

Proof. Suppose $(\eta, \varphi) - \mathscr{J}_W - \lim_{k\to\infty} \mathscr{C}_k = \mathscr{C}$. Then, for every $0 < \epsilon < 1$, for all $s > 0$ and $x \in X$, the set

$$\begin{aligned} U(\epsilon, s) = \{k \in \mathbb{N} : &|\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| \\ &\leq 1 - \epsilon \text{ or } | \varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| \geq \epsilon\} \end{aligned} \tag{23}$$

belongs to $\mathscr{J}$. Since $\mathscr{J}$ is an admissible ideal, then there exists $k_0 \in \mathbb{N}$ with the result that $k_0 \notin U(\epsilon, s)$. Now, suppose that

$$\begin{aligned} V(\epsilon, s) = \Big\{k \in \mathbb{N} : &|\eta(x, \mathscr{C}_k, s) - \eta\left(x, \mathscr{C}_{k_0}, s\right)| \\ &\leq (1 - 2\epsilon) \text{ or } | \varphi(x, \mathscr{C}_k, s) - \varphi\left(x, \mathscr{C}_{k_0}, s\right)| \geq 2\epsilon\Big\}. \end{aligned} \tag{24}$$

Considering the inequality

$$\begin{aligned} \left|\eta(x, \mathscr{C}_k, s) - \eta\left(x, \mathscr{C}_{k_0}, s\right)\right| &\leq |\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| \\ &+ \left|\eta\left(x, \mathscr{C}_{k_0}, s\right) - \eta(x, \mathscr{C}, s)\right|, \end{aligned} \tag{25}$$

and

$$\begin{aligned} |\varphi(x, \mathscr{C}_k, s) - \varphi\left(x, \mathscr{C}_{k_0}, s\right)| &\leq |\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| \\ &+ \left|\varphi\left(x, \mathscr{C}_{k_0}, s\right) - \varphi(x, \mathscr{C}, s)\right|. \end{aligned} \tag{26}$$

Observe that if $k \in V(\epsilon, s)$, therefore

$$|\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| + \left|\eta\left(x, \mathscr{C}_{k_0}, s\right) - \eta(x, \mathscr{C}, s)\right| \leq (1 - 2\epsilon), \tag{27}$$

and

$$|\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| + |\varphi\left(x, \mathscr{C}_{k_0}, s\right) - \varphi(x, \mathscr{C}, s)| \geq 2\epsilon. \tag{28}$$

From another point of view, since $k_0 \notin U(\epsilon, s)$, we obtain

$$\left|\eta\left(x, \mathscr{C}_{k_0}, s\right) - \eta(x, \mathscr{C}, s)\right| > 1 - \epsilon \text{ and } |\varphi\left(x, \mathscr{C}_{k_0}, s\right) - \varphi(x, \mathscr{C}, s)| < \epsilon. \tag{29}$$

We achieve that

$$|\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| \leq 1 - \epsilon \text{ or } |\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| \geq \epsilon. \tag{30}$$

Hence, $k \in U(\epsilon, s)$. This implies that $U(\epsilon, s) \subset V(\epsilon, s) \in \mathscr{J}$ for every $0 < \epsilon < 1$ and for all $s > 0$ and $x \in \mathscr{M}$. Therefore, $V(\epsilon, s) \in \mathscr{J}$, so the sequence is $\{\mathscr{C}_k\}$ which is Wijsman $\mathscr{J}$–Cauchy.

Theorem 25. *Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS and let $\mathscr{J}$ be an admissible ideal. Then, every Wijsman $\mathscr{J}^*$–Cauchy sequence of closed sets is Wijsman $\mathscr{J}$–Cauchy.*

Proof. Suppose that sequence $\{\mathscr{C}_k\}$ is Wijsman $\mathscr{J}^*$–Cauchy with respect to IFM (η, φ). Then, for each $x \in \mathscr{M}$ and for each $0 < \epsilon < 1$, there exists $P \in \mathscr{F}(\mathscr{J})$, where $P = \{(p_j) \colon p_j < p_{j+1}, j \in \mathbb{N}\}$ in such a way that

$$\left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta\left(x, \mathscr{C}_{p_l}, s\right)\right| \leq 1 - \epsilon, \tag{31}$$

and

$$\begin{gathered} \left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi\left(x, \mathscr{C}_{p_l}, s\right)\right| \geq \epsilon, \\ \forall k, l > k_0 = k_0(\epsilon). \end{gathered} \tag{32}$$

Suppose $N = N(\epsilon) = p_{k_0+1}$. Therefore, for each $\epsilon > 0$, one obtains

$$\left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta(x, \mathscr{C}_N, s)\right| \leq 1 - \epsilon, \tag{33}$$

and

$$\left|\varphi\left(x,\mathscr{C}_{p_k},s\right)-\varphi(x,\mathscr{C}_N,s)\right|\geq\epsilon\text{ for all }k>k_0. \tag{34}$$

Now, suppose that $K=\mathbb{N}\setminus P$. Clearly, $K\in\mathscr{J}$ and

$$\begin{aligned}Q(\epsilon,s)=\{k\in\mathbb{N}\mid \eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_N,s)|\leq 1\\ -\epsilon\text{ or }|\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_N,s)|\\ \geq\epsilon\subset K\cup\left\{p_1,p_2,\cdots,p_{k_0}\right\}\in\mathscr{J}.\end{aligned} \tag{35}$$

Hence, for all $s>0$ and for each $0<\epsilon<1$, one can determine $N=N(\epsilon)$ so that $Q(\epsilon,s)\in\mathscr{J}$, that is, sequence $\{\mathscr{C}_k\}$ is Wijsman $\mathscr{J}$–Cauchy.

Theorem 26. *Let $\mathscr{J}$ be an admissible ideal including property (AP) and $(\mathscr{M},\eta,\varphi,*,\diamond)$ be a separable IFMS. Then, the notion of Wijsman $\mathscr{J}^*$–Cauchy sequence of sets coincides with Wijsman $\mathscr{J}$ –Cauchy with respect to (η,φ) and vice-versa.*

Proof. The direct part is already proven in Theorem 25.

Now, suppose that sequence $\{\mathscr{C}_k\}$ is Wijsman $\mathscr{J}$–Cauchy sequence with respect to IFM (η,φ). Then by definition, if for every $0<\epsilon<1$, for each $x\in X$ and for all $s>0$, there exists a $m=m(\epsilon)$ such that

$$\begin{aligned}B(\epsilon,s)=\{k\in\mathbb{N}|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_n,s)|\\ \leq 1-\epsilon\text{ or }|\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_n,s)|\geq\epsilon\}\in I.\end{aligned} \tag{36}$$

Now, suppose that

$$\begin{aligned}P_j(\epsilon,s)=\left\{k\in\mathbb{N}\middle|\eta(x,\mathscr{C}_k,s)-\eta\left(x,\mathscr{C}_{m_j},s\right)\right|\\ >1-\frac{1}{j}\text{ or }|\,(x,\mathscr{C}_k,s)-\varphi\left(x,\mathscr{C}_{m_j},s\right)|<\left.\frac{1}{j}\right\},\end{aligned} \tag{37}$$

where $m_j=m(1/j)$, $j=1,2,3,\cdots$. Obviously, for $j=1,2,3\cdots$, $P_j(\epsilon,s)\in\mathscr{F}(\mathscr{J})$. Using Lemma 6, there exists $P\subset\mathbb{N}$ so that $P\in\mathscr{F}(\mathscr{J})$ and $P\setminus P_j$ are finite for all j.

Now, we prove that

$$\lim_{k,l\to\infty}|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_l,s)|=1, \tag{38}$$

and

$$\lim_{k,l\to\infty}|\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_l,s)|=0. \tag{39}$$

To show the above equations, let $\epsilon>0$, and $r\in\mathbb{N}$ such that $r>2/\epsilon$. If $k,l\in P$, then $P\setminus P_j$ is a finite set; therefore, there exists $w=w(r)$ in order that

$$\begin{aligned}\left|\eta(x,\mathscr{C}_k,s)-\eta\left(x,\mathscr{C}_{l_r},s\right)\right|>1-\frac{1}{r},\\ \left|\eta(x,\mathscr{C}_l,s)-\eta\left(x,\mathscr{C}_{l_r},s\right)\right|>1-\frac{1}{r},\end{aligned} \tag{40}$$

and

$$\begin{aligned}\left|\varphi(x,\mathscr{C}_k,s)-\varphi\left(x,\mathscr{C}_{l_r},s\right)\right|<\frac{1}{r},\\ \left|\varphi(x,\mathscr{C}_l,s)-\varphi\left(x,\mathscr{C}_{l_r},s\right)\right|<\frac{1}{r},\end{aligned} \tag{41}$$

for all $k,l>w(r)$. Then, the above inequalities follow that for $k,l>w(r)$

$$\begin{aligned}|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_l,s)|\leq\left|\eta(x,\mathscr{C}_k,s)-\eta\left(x,\mathscr{C}_{l_r},s\right)\right|\\ +\left|\eta(x,\mathscr{C}_l,s)-\eta\left(x,\mathscr{C}_{l_r},s\right)\right|>\left(1-\frac{1}{r}\right)+\left(1-\frac{1}{r}\right)>1-\epsilon,\end{aligned} \tag{42}$$

and

$$\begin{aligned}|\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_l,s)|\leq\left|\varphi(x,\mathscr{C}_k,s)-\varphi\left(x,\mathscr{C}_{l_j},s\right)\right|\\ +\left|\varphi(x,\mathscr{C}_l,s)-\varphi\left(x,\mathscr{C}_{l_j},s\right)\right|<\frac{1}{r}+\frac{1}{r}<\epsilon.\end{aligned} \tag{43}$$

Therefore, for each $\epsilon>0$, $\exists w=w(\epsilon)$ and $k,l\in P\in\mathscr{F}(I)$, we achieve

$$\begin{aligned}\{k\in\mathbb{N}:|\eta(x,\mathscr{C}_k,s)-\eta(x,\mathscr{C}_l,s)|\leq 1\\ -\epsilon\text{ or }|\,\varphi(x,\mathscr{C}_k,s)-\varphi(x,\mathscr{C}_l,s)|\geq\epsilon\}\in\mathscr{J}.\end{aligned} \tag{44}$$

This proves that the sequence $\{\mathscr{C}_k\}$ is a Wijsman $\mathscr{J}^*$–Cauchy.

Theorem 27. *Let $(\mathscr{M},\eta,\varphi,*,\diamond)$ be a separable IFMS and let $\mathscr{J}$ be an admissible ideal. Then*

$$(\eta,\varphi)-\mathscr{J}_W^*-\lim_{k\to\infty}\mathscr{C}_k=\mathscr{C} \tag{45}$$

implies that sequence $\{\mathscr{C}_k\}$ is a Wijsman $\mathscr{J}$–Cauchy sequence with respect to IFM (η,φ).

Proof. Suppose that $(\eta,\varphi)-\mathscr{J}_W^*-\lim_{k\to\infty}\mathscr{C}_k=\mathscr{C}$. Then, there exists $P=\{p=(p_j)\colon p_j<p_{j+1}, j\in\mathbb{N}\}\subset\mathbb{N}$ with $P\in\mathscr{F}(\mathscr{J})$ so that $\mathscr{C}_P=\{\mathscr{C}_{p_k}\}$

$$\lim_{k\to\infty}\eta\left(x,\mathscr{C}_{p_k},s\right)=\eta(x,\mathscr{C},s), \tag{46}$$

and

$$\lim_{k\to\infty} \varphi\left(x, \mathscr{C}_{p_k}, s\right) = \varphi(x, \mathscr{C}, s), \tag{47}$$

for any $\epsilon > 0$ and $k, l > k_0$.

Suppose $r \in \mathbb{N}$ and $\epsilon > 0$ in such a way that $r > 2/\epsilon$. If k, $l \in P$, then $P \setminus P_j$ is a finite set; therefore, there exists $k(r) = k$ so that

$$\begin{aligned} &\left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta\left(x, \mathscr{C}_{p_l}, s\right)\right| \le \left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - (x, \mathscr{C}, s)\right| \\ &\quad + \left|\eta\left(x, \mathscr{C}_{p_l}, s\right) - \eta(x, \mathscr{C}, s)\right| > \left(1 - \frac{1}{r}\right) + \left(1 - \frac{1}{r}\right) > 1 - \epsilon, \end{aligned} \tag{48}$$

and

$$\begin{aligned} &\left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi\left(x, \mathscr{C}_{p_l}, s\right)\right| < \left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi(x, \mathscr{C}, s)\right| \\ &\quad + \left|\varphi\left(x, \mathscr{C}_{p_l}, s\right) - \varphi(x, \mathscr{C}, s)\right| < \frac{1}{r} + \frac{1}{r} < \epsilon. \end{aligned} \tag{49}$$

Therefore,

$$\lim_{k,l\to\infty} \left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta\left(x, \mathscr{C}_{p_l}, s\right)\right| = 1, \tag{50}$$

and

$$\lim_{k,l\to\infty} \left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi\left(x, \mathscr{C}_{p_l}, s\right)\right| = 0. \tag{51}$$

Hence, sequence $\{\mathscr{C}_k\}$ is Wijsman $\mathscr{J}$–Cauchy with respect to IFM (η, φ).

4. Wijsman $\mathscr{J}$–cluster points and Wijsman $\mathscr{J}$–limit points in IFMS

Throughout this section, we denote $\mathscr{J}$ to be the proper ideal in $\mathbb{N}$ and define Wijsman $\mathscr{J}$–cluster and $\mathscr{J}$–limit points of the sequence of sets in intuitionistic fuzzy metric space and obtain some results.

Definition 28. Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS. An element $\mathscr{C} \in \mathscr{M}$ is known as the Wijsman $\mathscr{J}$ -cluster point of $\{\mathscr{C}_k\}$ if and only if for any $x \in \mathscr{M}$ and for all $\epsilon, s > 0$, one has

$$\begin{aligned} &\{k \in \mathbb{N} : |\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| < 1 \\ &- \epsilon \text{ or } |\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| > \epsilon\} \notin \mathscr{J}. \end{aligned} \tag{52}$$

We denote $\mathscr{J}_W^{(\eta,\varphi)}(\Gamma_{\{\mathscr{C}_k\}})$ as the collection of all Wijsman $\mathscr{J}$-cluster points.

Definition 29. Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS. An element $\mathscr{C} \in \mathscr{M}$ is known as Wijsman $\mathscr{J}$–limit point of sequence $\{\mathscr{C}_k\}$ of nonempty closed subsets of $\mathscr{M}$ provided that $P = \{p = (p_j) : p_j < p_{j+1}, j \in \mathbb{N}\} \subset \mathbb{N}$ in such a way that $P \notin \mathscr{J}$, and for any $x \in \mathscr{M}$ and $s > 0$, we obtain

$$\lim_{k\to\infty} \eta(x, \mathscr{C}_k, s) = \eta(x, \mathscr{C}, s) \text{ and } \lim_{k\to\infty} \varphi(x, \mathscr{C}_k, s) = \varphi(x, \mathscr{C}, s). \tag{53}$$

We denote $\mathscr{J}_W^{(\eta,\varphi)}(\Lambda_{\{\mathscr{C}_k\}})$ as the collection of all Wijsman $\mathscr{J}$–limit points.

Theorem 30. *Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS. Then, for any sequence sets, $\{\mathscr{C}_k\} \subset \mathscr{M}$, $\mathscr{J}_W^{(\eta,\varphi)}(\Lambda_{\{\mathscr{C}_k\}}) \subset \mathscr{J}_W^{(\eta,\varphi)}(\Gamma_{\{\mathscr{C}_k\}})$.*

Proof. Suppose $\mathscr{C} \in \mathscr{J}_W^{(\eta,\varphi)}(\Lambda_{\{\mathscr{C}_k\}})$. Then, there exists $P = \{p_1 < p_2 < \cdots\} \subset \mathbb{N}$ such that $P = \{p = (p_j) : p_j < p_{j+1}, j \in \mathbb{N}\} \notin \mathscr{J}$ and for all $s > 0$ and $x \in \mathscr{M}$, we have

$$\lim_{k\to\infty} \eta\left(x, \mathscr{C}_{p_k}, s\right) = \eta(x, \mathscr{C}, s), \tag{54}$$

and

$$\lim_{k\to\infty} \varphi\left(x, \mathscr{C}_{p_k}, s\right) = \varphi(x, \mathscr{C}, s). \tag{55}$$

According to Equations (54) and (55), there exists $k_0 \in \mathbb{N}$ so that for each $\epsilon > 0$ and for any $x \in X$ and $k > k_0$

$$\left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta(x, \mathscr{C}, s)\right| > 1 - \epsilon, \tag{56}$$

and

$$\left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi(x, \mathscr{C}, s)\right| < \epsilon. \tag{57}$$

Hence,

$$\begin{aligned} &\left\{k \in \mathbb{N} : \left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta(x, \mathscr{C}, s)\right| > 1 \right. \\ &\left. - \epsilon, \left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi(x, \mathscr{C}, s)\right| < \epsilon\right\} \supseteq \left\{P \setminus p_1, p_1, ; p_{k_0}\right\}. \end{aligned} \tag{58}$$

Then, the right-hand side of (58) does not belong to $\mathscr{J}$, and then

$$\begin{aligned} &\left\{k \in \mathbb{N} : \left|\eta\left(x, \mathscr{C}_{p_k}, s\right) - \eta(x, \mathscr{C}, s)\right| > 1 \right. \\ &\left. - \epsilon, \left|\varphi\left(x, \mathscr{C}_{p_k}, s\right) - \varphi(x, \mathscr{C}, s)\right| < \epsilon\right\} \notin \mathscr{J}, \end{aligned} \tag{59}$$

which means that $\mathscr{C} \in \mathscr{J}_W^{(\eta,\varphi)}(\Gamma_{\{\mathscr{C}_k\}})$.

Theorem 31. *Let $(\mathscr{M}, \eta, \varphi, *, \diamond)$ be a separable IFMS. Then, for any sequence $\{\mathscr{C}_k\} \subset \mathscr{M}$, $\mathscr{J}_W^{(\eta,\varphi)}(\Gamma_{\{\mathscr{C}_k\}}) \subset L_{\{\mathscr{C}_k\}}$.*

Proof. Let $\mathscr{C} \in \mathscr{I}_W^{(\eta,\varphi)}(\Gamma_{\{\mathscr{C}_k\}})$. Then, for each $\epsilon > 0$ and for all $s > 0$ and for each $x \in \mathscr{M}$, one has

$$\{k \in \mathbb{N} : |\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| < 1 - \epsilon \text{ or } |\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| > \epsilon\} \notin \mathscr{I}. \tag{60}$$

Suppose

$$Q_k = \left\{k \in \mathbb{N} : |\eta(x, \mathscr{C}_k, s) - \eta(x, \mathscr{C}, s)| > 1 - \frac{1}{k}, |\varphi(x, \mathscr{C}_k, s) - \varphi(x, \mathscr{C}, s)| < \frac{1}{k}\right\}, \tag{61}$$

for $k \in \mathbb{N}$. $\{Q_k\}_{k=1}^{\infty}$ is a descending sequence of subsets of $\mathbb{N}$. Hence, $Q = \{k = (k_i) \colon k_i < k_{i+1}, i \in \mathbb{N}\} \notin \mathscr{I}$ so that

$$\lim_{k\to\infty} \eta\left(x, \mathscr{C}_{k_i}, s\right) = \eta(x, \mathscr{C}, s), \tag{62}$$

and

$$\lim_{k\to\infty} \varphi\left(x, \mathscr{C}_{k_i}, s\right) = \varphi(x, \mathscr{C}, s), \tag{63}$$

which means that $\mathscr{C} \in L_{\{\mathscr{C}_k\}}$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this article.

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