NATURENGS
MTU Journal of Engineering and Natural Sciences
https://dergipark.org.tr/tr/pub/naturengs
DOI: 10.46572/naturengs.882203

Research Article

A Computational Method Based on Interval Length for Fuzzy Time Series Forecasting

Özlem AKAY

Department of Statistics, Faculty of Science and Letters, Çukurova University, Adana, Turkey

(**Received:** 17.02.2021; **Accepted:** 26.03.2021)

ABSTRACT: In the literature, there have been a good many different forecasting methods related to forecasting problems of fuzzy time series. The main issue of fuzzy time series forecasting is the accuracy of the forecasted values. The forecasting accuracy rate is affected by the length of each interval in the universe of discourse. Thus, it is substantial to determine the length of each interval. In this study, a new computational method based on class width to determine interval length is proposed and also used the coefficient of variation for time series forecasting. After the intervals are formed, the historical time series data set is fuzzified according to fuzzy time series theory. The proposed model has been tested on the student enrollments, University of Alabama, and a real-life problem of rice production for containing higher uncertainty. This method was compared with existent methods to determine the effectiveness in terms of the mean square error (MSE) and the average forecasting (AFE). The results are shown that the proposed model can achieve a higher forecasting accuracy rate than the existing models.

Keywords: Interval length, Coefficient of variation, Fuzzified, Fuzzy time series, Forecasting.

1. INTRODUCTION

Forecasting activities are very important in daily life. If things such as earthquakes, stock markets, weather can be predicted by people in advance, people take the necessary precautions and make their plans according to the situation that will occur [1]. Regression analysis and moving average methods are constantly used for forecasting [2]. However, these methods are not attending the forecasting problems in which the historical data are in linguistic terms [3]. Zadeh (1965) [4] developed the concept of linguistic variables, fuzzy set theory, and its application to approximate reasoning. Song and Chissom (1993) [5] have successfully employed this in fuzzy time series forecasting. Many researchers have recommended various forecasting methods based on fuzzy time series to cope with forecasting problems, in recent years. Fuzzy time series forecasting appeared as a method for predicting the future values in a situation in which neither a pattern in variations of time series are visualized nor a trend is viewed and also the information is vague and imprecise [5].

Hwang et al. (1998) [1] suggested a model based on time-variant fuzzy time series to cope with the forecasting problems. The opinion of this model is that the variation of enrollment of this year is related to the trend of the enrollments of the past years. Huarng (2001) [6] recommended

Corresponding Author: oakay@cu.edu.tr
ORCID number of authors: 0000-0002-9539-7202

heuristic models to improve forecasting by integrating problem-specific heuristic knowledge with Chen's model. Huarng (2001) [7] proposed distribution-and average-based length to approach the issue of how to determine effective lengths of intervals. Distribution-based length is the largest length smaller than at least half the first differences of data. The average-based length is set to one-half the average of the first differences of data. Singh (2007) [8] offered a new method which is a simplified computational approach for fuzzy time series forecasting. The new model is included different parameters. Singh (2007) [9] recommended a simple time-variant model by using the difference operator for time series forecasting. Values obtained have been used for developing fuzzy rules. Sing (2007) [10] presented a new method in a form of simple computational algorithms. This method is a versatile method of forecasting based on the concept of fuzzy time series. Singh (2008) [3] improved a model based on fuzzy time series having the ability to deal with the situation of high uncertainty having large fluctuations in the consecutive values. Yolcu et al. (2009) [11] proposed a new approach that uses a single-variable constrained optimization to determine the ratio for the length of intervals. This approach was applied to the two well-known time series, which are enrollment data at The University of Alabama and inventory demand data. The obtained results were compared to those of other methods. The proposed method produced more accurate predictions for the future values of the used time series. Gangwar and Kumar (2012) [2] proposed a new model to enhance the accuracy in forecasted values. This method is based on multiple partitioning and higher-order fuzzy time series. Singh and Borah (2013) [12] presented a new model for addressing four main issues in fuzzy time series forecasting. These are defuzzification of fuzzified time series values, handling of fuzzy logical relationship (FLR), determination of weight for each FLR, and determining the effective length of intervals. The determination of the length of intervals is very important for the fuzzification of the time series data set. The length of the intervals was kept the same in most of the fuzzy time series models. There is no specific reason for using fixed range intervals. Huarng (2001) [6] indicates that the effective length of interval always affects the results of forecasting [12]. Wang et al. (2013) [13] studied how to partition the universe of discourse into intervals with unequal length to improve forecasting quality. They calculated the prototypes of data using fuzzy clustering, then formed some subsets according to the prototypes, and proposed an unequal length partitioning method. Ye et al. (2016) [14] proposed a new forecasting method based on multi-order fuzzy time series, technical analysis, and a genetic algorithm. In this algorithm, multi-order fuzzy time series (first-order, second-order, and third-order) are applied and a genetic algorithm is used to find a good domain partition. Yolcu et al. (2016) [15] proposed a novel high-order fuzzy time series approach that considers the membership values, where artificial neural networks are employed to identify the fuzzy relations. In the proposed method, intersection operators are utilized to deal with an excessive number of inputs, and also the fuzzy c-means method is employed for fuzzification. Jiang et al. (2017) [16] constructed a novel high-order fuzzy time series (FTS) model to make time series forecasting. In this study, the optimal lengths of intervals were tuned by applying the harmony search intelligence algorithm. Regularly increasing monotonic quantifiers were employed on fuzzy sets to obtain the weights of ordered weighted aggregation. Singh (2017) [17] introduced a new fuzzy time series (FTS) forecasting model. In this model, for the determination of effective lengths of intervals, the "frequency-based discretization" approach was proposed, which partitions the time series data set into various lengths. Besides the establishment of the fuzzy logical relations (FLRs) and their better representation, an artificial neural network-based architecture was developed. Chen and Phuong (2017) [18] proposed a new fuzzy time series (FTS) forecasting method based on optimal partitions of intervals in the universe of discourse and optimal weighting vectors of two-factors second-order fuzzy-trend logical relationship groups (TSFTLRGs). This method uses particle swarm optimization (PSO) techniques to obtain the optimal partitions of intervals and the optimal weighting vectors simultaneously. Bas et al. (2018) [19] introduced a new model for determining the fuzzy relationships for high order fuzzy time series forecasting which uses Pi-Sigma neural network. They used a modified particle swarm optimization model o train the Pi-Sigma network. Panigrahi and Behera (2020) [20] addressed two key issues such as modeling fuzzy logical relationships (FLRs) and determination of the effective length of the interval. They used machine learning (ML) techniques for modeling the FLRs and proposed a modified average-based method to estimate the effective length of the interval. Pattanayak et al. (2020) [21] developed a novel method using fuzzy c-means clustering to determine the unequal length of the interval. The membership values were considered while modeling the fuzzy logical relationships using a support vector machine (SVM). The order of the model is determined by analyzing the autocorrelation function and partial autocorrelation function of the time series.

The objective of this study is to solve the problem of determining the length of the intervals by adjusting the new length of the intervals to obtain more accurate forecasting. So, a computational model forecasting based on multiple partitioning is presented in this paper. The proposed method presents simple computational algorithms and is easier for the application. The algorithm of proposed methods has been performed for forecasting the enrollments of the University of Alabama. To display the superiority of this method, the results obtained have been compared with the existent methods. Further, the model suitability in the general forecasting problem has been performed in another real-life dataset. Therefore the proposed method has been applied to the time series data of rice production. This data set is obtained from Pantnagar farm, G.B. Pant University of, Agriculture and Technology, India.

The paper is organized as follows. The fuzzy time series are briefly introduced and related works for fuzzy time series models are presented in section 1. In section 2, an overview of the fuzzy time series has been explained. A new model based on determining the length of the interval is presented for forecasting enrollments in section 3. In section 4, the algorithm of the proposed method was applied to two different data sets, and the mean square error (MSE) and the average forecasting error (AFE) are computed to compare with the existing methods. The conclusions are discussed in section 5.

2. MATERIAL AND METHODS

2.1. Fuzzy Time Series

Song and Chissom (1993) [5] recommended the definitions of fuzzy time series. The various properties and definitions of fuzzy time series forecasting are given stepwise as follows;

1. A fuzzy set is a class of objects with a continuum of a grade of membership. C is the Universe of discourse with $C = \{c_1, c_2, c_3, \dots, c_n\}$, where c_i are possible linguistic values of C, a fuzzy set of linguistic variables K_i of C is described by

$$K_i = \frac{\mu_{K_i}(c_1)}{c_1} + \frac{\mu_{K_i}(c_2)}{c_2} + \frac{\mu_{K_i}(c_3)}{c_3} + \cdots + \frac{\mu_{K_i}(c_n)}{c_n}$$

here $\mu_{K_i}$is the membership function of the fuzzy set K_i, $\mu_{K_i}: C = [0,1]$.

If c_j is the member of K_i, $\mu_{K_i}(c_j)$ is the degree of belonging of c_j to K_i.

2. *Y(t)* (t=…,0,1,2,3,…), is a subset of R, the universe of discourse on which fuzzy sets $f_i(t)$ (*i*=1,2,3,…) are described. *F(t)* is the collection of f_i, and *F(t)* is described as fuzzy time series on *Y(t).*

3. Assume that there is a fuzzy relationship between *F(t)* and *F(t-1).* If *F(t-1)* only causes *F(t),* $F(t-1) \rightarrow F(t)$ is demonstrated. The fuzzy relational equation expressed as:

F(t)=F(t-1)oR(t, t-1)

where "o" is the max-min composition operator. The relation *R* is called the first-order model of *F(t).* Further, if fuzzy relation *R(t,t-1)* of *F(t)* is independent of time *t,* that means $R(t_1, t_1\text{-}1) = R(t_2, t_2\text{-}1)$ for different times t_1 and t_2, then *F(t)* is called a time-invariant fuzzy time series

4. Assume that *F(t)* is caused by more fuzzy sets, *F(t-n), F(t-n+1),…, F(t-1).* The fuzzy relationship is demonstrated by

$$K_{i1}, K_{i2}, \dots, K_{in} \rightarrow K_j$$

where $F(t-n) = K_{i1}$, $F(t-n+1), = K_{i2}$, …, $F(t-1) = K_{in}$. This relationship is called the *n*th order fuzzy time series model.

5. Assume that an *F(t-1), F(t-2),…,* and *F(t-m) (m>0)* simultaneously causes *F(t).* The *F(t)* is said to be time-variant fuzzy time series. The fuzzy relational equation expressed as:

$F(t)=F(t-1)oR^{w}(t,t-1)$

here *w>1* is several years (time) parameter by which the forecast *F(t)* is being affected. A variety of complex calculation methods are existing for the calculations of the relation $R^{w}(t, t\text{-}1)$.

2.2. Proposed Method

In this part, the algorithm suggested by Song and Chissom (1993) [5] was modified. The class width was used to determine interval length and the coefficient of variation was used for time series forecasting. The algorithm of the proposed computational method is presented stepwise.

1. Determine the Universe of discourse, *C* based on the range of time-series data

$$R = E_{max} - E_{min} \tag{1}$$

$$m = 2\sqrt{n} \tag{2}$$

$$h = \frac{R}{m} \tag{3}$$

Intervals are established by continuing to add the class width (*h*) to the minimum data values until the number of classes (*m*) is created.

$$C = [E_{\min}, E_{\min} + h] \tag{4}$$

2. Divide the Universe of discourse into an equal length of intervals: $c_1, c_2, \dots, c_m$. The number of intervals will be according to the number of linguistic variables (fuzzy sets) $K_1, K_2, \dots, K_m$ to be noted.

3. According to the intervals in Step 2 configure linguistic variables K_i and perform the triangular membership rule to each interval in each created fuzzy set.

4. After this time series data is fuzzified, the fuzzy logical relationships are stated. *I*f K_i is the fuzzy production of year *n* and K_j is the fuzzify production of year *n+1*, the fuzzy logical relation is demonstrated as $K_i \rightarrow K_j$, where K_i is the current case and K_j is the next case.

5. Algorithm

Used notations are shown as;

[$*K_j$] is the corresponding interval c_j for which membership in K_j is the supremum

M[$*K_j$] is the mid-value of the interval c_j having supremum value in K_j

U[$*K_j$] is the upper bound of the interval [$*K_j$]

L[$*K_j$] is the lower bound of the interval [$*K_j$]

For $K_i \rightarrow K_j$ (a fuzzy logical relation)

E_i is the actual production of year *n*

E_{i-1} is the actual production of year *n-1*

E_{i-2} is the actual production of year *n-2*

F_j is the crisp forecasted production of the years *n+1*
Algorithm for forecasting production of year *n+1* and onwards
For *t=3…T*

Attained fuzzy logical relation for year *t* to *t+1*

$$K_i \rightarrow K_j$$

$$ED_i = |(E_i - E_{i-1})| \tag{5}$$

$$ED_{ii} = |(E_{i-1} - E_{i-2})| \tag{6}$$

$$\overline{ED} = \frac{ED_i + ED_{ii}}{2} \tag{7}$$

$$S_i = (ED_i - \overline{ED}) + (ED_{ii} - \overline{ED}) \tag{8}$$

$$X_i = \mathrm{M}[* K_j] + \frac{S_i * 100}{\overline{ED}} \tag{9}$$

$$F_j = X_i \tag{10}$$

Next *t*

3. RESULTS AND DISCUSSION

The algorithm of the proposed approach mentioned above is applied in the time series data of the records in the University of Alabama. The steps of the algorithm and the results obtained are as follows;

Step 1. Based on the proposed method, the Universe of discourse is divided into equal lengths of intervals. In Table 1, the results obtained are presented. Each interval is computed by using Eq. (1-4) respectively as below;

R=19337-13055=6282

$$m = 2\sqrt{22} \cong 10$$

$$h = \frac{6282}{10} = 628.2 \cong 629$$

$c_1 = [13055, 13684]$, $c_2 = [13684, 14313]$, $c_3 = [14313, 14942]$, $c_4 = [14942, 15571]$,

$c_5 = [15571, 16200]$, $c_6 = [16200, 16829]$, $c_7 = [16829, 17458]$,

$c_8 = [17458{,}18087]$, $c_9 = [18087, 18716]$, $c_{10} = [18716, 19345]$

Mid-point is stated for each interval and stocked for future consideration.

Table 1. Obtained new intervals, mid-points, and their corresponding elements.

Intervals	Boundaries	Mid-point	Corresponding element
c_1	$[13055, 13684]$	13369.5	13055, 13563
c_2	$[13684, 14313]$	13998.5	13867
c_3	$[14313, 14942]$	14627.5	14696
c_4	$[14942, 15571]$	15256.5	15145, 15163, 15311,15433, 15460, 15497
c_5	$[15571, 16200]$	15885.5	15603, 15861, 15984
c_6	$[16200, 16829]$	16514.5	16388, 16807
c_7	$[16829, 17458]$	17143.5	16859, 16919
c_8	$[17458{,}18087]$	17772.5	
c_9	$[18087, 18716]$	18401,5	18150
c_{10}	$[18716, 19345]$	19030.5	18876,18970, 19328, 19337

Step 2. For each of the interval, described linguistic terms

This time-series data set is partitioned ten intervals ($c_1, c_2, \dots, c_{10}$). Therefore, a total of 10 linguistic variables ($K_1, K_2, \dots, K_{10}$) are described. Fuzzy sets demonstrate all these linguistic variables as follows:

$$K_1 = \left\{\frac{1}{c_1} + \frac{0.5}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0}{\square_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_2 = \left\{\frac{0.5}{c_1} + \frac{1}{c_2} + \frac{0.5}{c_3} + \frac{0}{c_4} + \frac{0}{c_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_3 = \left\{\frac{0}{c_1} + \frac{0.5}{c_2} + \frac{1}{c_3} + \frac{0.5}{c_4} + \frac{0}{c_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_4 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0.5}{c_3} + \frac{1}{c_4} + \frac{0.5}{c_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_5 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0.5}{c_4} + \frac{1}{c_5} + \frac{0.5}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_6 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0.5}{c_5} + \frac{1}{c_6} + \frac{0.5}{c_7} + \frac{0}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_7 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0}{c_5} + \frac{0.5}{c_6} + \frac{1}{c_7} + \frac{0.5}{c_8} + \frac{0}{c_9} + \frac{0}{c_{10}}\right\}$$
$$K_8 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0}{c_5} + \frac{0}{c_6} + \frac{0.5}{c_7} + \frac{1}{c_8} + \frac{0.5}{c_9} + \frac{0}{c_{10}}\right\}$$

$$K_9 = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0}{c_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0.5}{c_8} + \frac{1}{c_9} + \frac{0.5}{c_{10}}\right\}$$
$$K_{10} = \left\{\frac{0}{c_1} + \frac{0}{c_2} + \frac{0}{c_3} + \frac{0}{c_4} + \frac{0}{c_5} + \frac{0}{c_6} + \frac{0}{c_7} + \frac{0}{c_8} + \frac{0.5}{c_9} + \frac{1}{c_{10}}\right\}$$

The degree of membership of each year's enrollment value belonging to each K_i is procured, where, the maximum degree of membership of a fuzzy set K_i forms at interval c_i $(1 \leq i \leq 10)$.

Step 3. Fuzzify this data set.

If one year's enrollment value belongs to the interval c_i, for that year, the fuzzified enrollment value is considered as K_i. For instance, since the enrollment value of the year 1971 belongs to the interval c_1, it is fuzzified to K_1. Thus, this data set is fuzzified. The values of actual and fuzzified enrollments are given in Table 2.

Table 2. Actual and fuzzified enrollments of University of Alabama

Year	Actual enrollments	Fuzzified enrollments
1971	13055	K_1
1972	13563	K_1
1973	13867	K_2
1974	14696	K_3
1975	15460	K_4
1976	15311	K_4
1977	15603	K_5
1978	15861	K_5
1979	16807	K_6
1980	16919	K_7
1981	16388	K_6
1982	15433	K_4
1983	15497	K_4
1984	15145	K_4
1985	15163	K_4
1986	15984	K_5
1987	16859	K_7
1988	18150	K_9
1989	18970	K_{10}
1990	19328	K_{10}
1991	19337	K_{10}
1992	18876	K_{10}

Step 4.

The calculations have been performed by Eq. (5-10) respectively for the proposed method. The results obtained along with the results of other methods are given in Table 3.

Forecasting for 14696;

$$ED_i = |13867 - 13563|$$

$$ED_{ii} = |13563 - 13055|$$

$$\overline{ED} = \frac{304 + 508}{2}$$

$$S_i = (304 - 406) + (508 - 406)$$

$$X_i = 14627.5 + \frac{144.249 * 100}{406}$$

$$F_j = 14663.029$$

MSE and AFE are used to evaluate the accuracy of fuzzy time series forecasting. For the forecasting method, lower values of MSE or AFE are better. The equations of AFE and MSE are as follows;

$$MSE = \frac{\sum_{i=1}^{n}(Actual\ value_i - Forecasted\ value_i)^2}{n}$$

$$AFE\ (in\ \%) = \frac{sum\ of\ forecasting\ errors}{n}$$

$$Forecasting\ errors\ (in\ \%) = \frac{|Forecasted\ value - Actual\ value|}{Actual\ value} x100$$

The MSE and AFE have been computed to compare the accuracy of the forecasted value obtained by the proposed method with other methods. In Table 3, the values of AFE and MSE calculated are given.

Table 3. Comparative presentations of enrollments forecast by various methods.

Year	**Actual enrollment**	**Method (Cheng et al. 2008) [22]**	**Method (Wong et al. 2010) [23]**	**Method (Chen and Tanuwijay, 2011) [24]**	**Method (Gangwar and Kumar, 2012) [2]**	**Method (Singh and Borah, 2013) [12]**	**Proposed Method**
1971	13055	-	-	-	-	-	-
1972	13563	14242	-	13512	-	13563	-
1973	13867	14242	13500	13998	13500	13867	-
1974	14696	14242	14500	14658	14500	14696	14663.029
1975	15460	15474.3	15500	15341	15500	15425	15322.030
1976	15311	15474.3	15466	15501	15500	15420	15262.270
1977	15603	15474.3	15392	15501	15500	15420	15980.76
1978	15861	15474.3	15549	15501	15500	15923	15931.357
1979	16807	16146.5	16433	17065	-	16862	16523.242
1980	16919	16988.3	16656	17159	-	17192	17224.31
1981	16388	16988.3	16624	17159	16500	17192	16625.979
1982	15433	16146.5	15556	15341	15500	15425	15348.654
1983	15497	15474.3	15524	15501	15500	15420	15296.851
1984	15145	15474.3	15497	15501	15500	15420	15380.156
1985	15163	15474.3	15305	15501	15500	15627	15354.407
1986	15984	15474.3	15308	15501	-	15627	16013.161
1987	16859	16146.5	16402	17065	-	16862	17278.853
1988	18150	16988.3	18500	17159	18500	17192	18406.002
1989	18970	19144	18534	18832	18500	18923	19057.661
1990	19328	19144	19345	19333	19337	19333	19062.053
1991	19337	19144	19423	19083	19500	19136	19085.964
1992	18876	19144	18752	19083	18704	19136	19164.985
MSE	-	228909.4	88337.2	122085	62976.63	106037.1	**53012.28**
AFE	-	2.3915	1.51894	1.53526	1.269981	**1.19016**	**1.19202**

As shown in Table 3, the MSE value of the proposed model is lower than that of the other models. AFE values of the proposed method and Singh and Borah's (2013) [12] method are very close to each other. According to the values of MSE and AFE, the proposed method provides a forecast of higher accuracy. This indicates the superiority of the proposed method over the others.

The sufficiency of the proposed model is also investigated by performing it into the real-life problem of a dynamic system containing fuzziness like rice production. For this reason, the time-series data of rice production obtained from the huge farm of G.B. Pant University, Pantnagar is used. This data is formed according to productivity in kg per hectare. The algorithm of the proposed model has been applied as above and the results obtained are offered.

R=4554-3219=1335

$$m = 2\sqrt{20} \cong 10$$

$$h = \frac{1335}{10} \cong 134$$

$c_1 = [3219, 3353]$, $c_2 = [3353, 3487]$, $c_3 = [3487, 3621]$, $c_4 = [3621, 3755]$,
$c_5 = [3755, 3889]$, $c_6 = [3889, 4023]$, $c_7 = [4023, 4157]$,
$c_8 = [4157, 4291]$, $c_9 = [4291, 4425]$, $c_{10} = [4425, 4559]$

Lengths of the interval were determined according to the proposed approach and mid-points were found for each interval. In Table 4, new intervals obtained, mid-points, and their corresponding elements are given.

Table 4. Obtained new intervals, mid-points and their corresponding elements.

Intervals	Mid-point	Corresponding elements
c_1	3286	3222, 3219
c_2	3420	3372
c_3	3554	3552, 3455, 3592,
c_4	3688	3702, 3670, 3750
c_5	3822	3865, 3851, 3872
c_6	3956	3928
c_7	4090	
c_8	4224	4177, 4170, 4266
c_9	4358	4305
c_{10}	4492	4554, 4439

This data set is split up ten intervals $(c_1, c_2, \dots, c_{10})$ and a total of 10 linguistic variables $(K_1, K_2, \dots, K_{10})$ are assigned. Similarly, the linguistic variables are indicated by fuzzy sets as shown in Step 2 and Step 3.

The degree of membership of each year's enrollment value belonging to each c_i is obtained and this time series data set is fuzzified. The calculations have been carried out by using the algorithm of the proposed method. The actual and fuzzified enrollments values and the results obtained are given in Table 5.

Table 5. Actual and Fuzzified enrollments of rice production

Year	Actual enrollments	Fuzzified enrollments	Proposed model
1981	3552	K_3	-
1982	4177	K_8	-
1983	3372	K_2	-
1984	3455	K_3	3571.801
1985	3702	K_4	3802.984
1986	3670	K_4	3758.281
1987	3865	K_5	3930.97
1988	3592	K_3	3668.688
1989	3222	K_1	3309.570
1990	3750	K_4	3709.334
1991	3851	K_5	3846.882
1992	3231	K_1	3382.004
1993	4170	K_8	4295.844
1994	4554	K_{10}	4520.937
1995	3872	K_5	3881.326
1996	4439	K_{10}	4531.534
1997	4266	K_8	4234.87
1998	3219	K_1	3361.297
1999	4305	K_9	4459.313
2000	3928	K_6	3958.585

MSE and AFE values were calculated to compare the proposed model with other models. The results are given in Table 6.

Table 6. MSE and AFE of rice production forecast

	Proposed Method	Chen's method (1996) [25]	Song and Chissom method (1993) [5]
MSE	**8561.948**	132162.9	131715.9
AFE	**2.148**	7.934	7.948

In Table 6, it is shown that the suitability of the proposed method is better than Chen's (1996) [25] model and the method of Song and Chissom (1993) [5] by the comparison of MSE and AFE.

4. CONCLUSIONS

In this study, a computation model with high accuracy based on the lengths of the interval is proposed. The algorithms of the proposed model are simple. This model has been used for forecasting the time series data of enrollments of the University of Alabama and has been compared with the existing models. MSE and AFE values were used to compare with the methods. AFE and MSE were used for the measurement of the accuracy of the forecast. According to the values of AFE and MSE obtained by methods, the proposed model outperforms the models proposed by Cheng et al. (2008) [22], Wong et al. (2010) [23], Chen and Tanuwijaya (2011) [24], Gangwar and Kumar (2012) [2], and Singh and Borah (2013) [12] in this dataset. Moreover, the proposed method has also been performed on rice production forecasts. The performance of the proposed method in the forecasting of the rice production is compared with the models suggested by Chen's method (1996) [25] and Song and Chissom method (1993) [5]. By MSE and AFE, the proposed model provides results of better accuracy than both models. Therefore, the proposed method is a preferable model for fuzzy time series forecasting.

REFERENCES

[1] Hwang, J.R., Chen, S.M. and Lee, C.H. (1998). Handling forecasting problems using fuzzy time series, *Fuzzy sets and systems*, 100(1-3): 217-228.

[2] Gangwar, S.S. and Kumar, S. (2012). Partitions based computational method for high-order fuzzy time series forecasting, *Expert Systems with Applications*, 39(15): 12158-12164.

[3] Singh, S.R. (2008). A computational method of forecasting based on fuzzy time series, *Mathematics and Computers in Simulation*, 79(3): 539-554.

[4] Zadeh, L. A. (1965). Fuzzy sets, *Information and control*, 8(3): 338-353.

[5] Song, Q. and Chissom, B.S. (1993). Forecasting enrollments with fuzzy time series—part I, *Fuzzy sets and systems*, 54(1): 1-9.

[6] Huarng, K. (2001). Heuristic models of fuzzy time series for forecasting, *Fuzzy sets and systems*, 123(3): 369-386.

[7] Huarng, K. (2001). Effective lengths of intervals to improve forecasting in fuzzy time series, *Fuzzy sets and systems,* 123(3): 387-394.

[8] Singh, S.R. (2007). A robust method of forecasting based on fuzzy time series, *Applied Mathematics and Computation*, 188(1): 472-484.

[9] Singh, S.R. (2007). A simple method of forecasting based on fuzzy time series, *Applied mathematics and computation,* 186(1): 330-339.

[10] Singh, S.R. (2007). A simple time variant method for fuzzy time series forecasting, *Cybernetics and Systems: An International Journal*, 38(3): 305-321.

[11] Yolcu, U., Egrioglu, E., Uslu, V. R., Basaran, M. A. and Aladag, C. H. (2009). A new approach for determining the length of intervals for fuzzy time series, *Applied Soft Computing,* 9(2): 647-651.

[12] Singh, P. and Borah, B. (2013). An efficient time series forecasting model based on fuzzy time series, *Engineering Applications of Artificial Intelligence*, 26(10): 2443-2457.

[13] Wang, L., Liu, X. and Pedrycz, W. (2013). Effective intervals determined by information granules to improve forecasting in fuzzy time series, *Expert Systems with Applications*, *40*(14): 5673-5679.

[14] Ye, F., Zhang, L., Zhang, D., Fujita, H. and Gong, Z. (2016). A novel forecasting method based on multi-order fuzzy time series and technical analysis, *Information Sciences*, *367*: 41-57.

[15] Yolcu, O. C., Yolcu, U., Egrioglu, E. and Aladag, C. H. (2016). High order fuzzy time series forecasting method based on an intersection operation, *Applied Mathematical Modelling*, *40*(19-20): 8750-8765.

[16] Jiang, P., Dong, Q., Li, P. and Lian, L. (2017). A novel high-order weighted fuzzy time series model and its application in nonlinear time series prediction, *Applied Soft Computing*, *55*: 44-62.

[17] Singh, P. (2017). High-order fuzzy-neuro-entropy integration-based expert system for time series forecasting, *Neural Computing and Applications*, *28*(12): 3851-3868.

[18] Chen, S. M. and Phuong, B. D. H. (2017). Fuzzy time series forecasting based on optimal partitions of intervals and optimal weighting vectors, *Knowledge-Based Systems*, *118*: 204-216.

[19] Bas, E., Grosan, C., Egrioglu, E. and Yolcu, U. (2018). High order fuzzy time series method based on pi-sigma neural network, *Engineering Applications of Artificial Intelligence*, *72*: 350-356.

[20] Panigrahi, S. and Behera, H. S. (2020). A study on leading machine learning techniques for high order fuzzy time series forecasting, *Engineering Applications of Artificial Intelligence*, *87:* 103245.

[21] Pattanayak, R. M., Panigrahi, S. and Behera, H. S. (2020). High-order fuzzy time series forecasting by using membership values along with Data and Support Vector Machine, *Arabian Journal for Science and Engineering*, *45*(12): 10311-10325.

[22] Cheng, C. H., Wang, J. W. and Cheng, G. W. (2008). Multi-attribute fuzzy time series method based on fuzzy clustering, Expert Systems with Applications, 34: 1235–1242.

[23] Wong, W.K., Bai, E. and Chu, A.W.C. (2010). Adaptive time-variant models for fuzzy-time-series forecasting, *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 40(6): 1531-1542.

[24] Chen, S.M. and Tanuwijaya, K. (2011). Fuzzy forecasting based on high-order fuzzy logical relationships and automatic clustering techniques, *Expert Systems with Applications*, 38(12): 15425-15437.

[25] Chen, S.M. (1996). Forecasting enrollments based on fuzzy time series, *Fuzzy sets and systems*, 81(3): 311-319.